Cost of Capital, Valuation and Strategic Financial Decision Making

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The financial crisis that hit financial markets in 2007 came at the end of a period characterised by abundant availability of cheap finance. Securing funding to finance capital expenditure and investment in working capital has since become more of a demanding task. It is therefore all the more important to understand in full the determinants of the cost of capital faced by a firm. In this article, we will discuss the extent to which the cost of capital depends on capital structure and illustrate the implications for valuation.

First, it is probably useful that we emphasize why it is useful to think about the cost of capital. A firm, in its business operations, uses capital. In a perfect capital market, the providers of capital would be able to ex ante identify and evaluate the use to which the capital provided would be put by the financed firms. This is the world hypothesised by Modigliani and Miller in their 1958 article. In such an idealised world, capital would be in unlimited supply and its cost, i.e. the expected return demanded by investors, would only depend on the risk of each project no matter how the project or the firm is financed.

This all changes when we allow for the ‘frictions’ of real world capital markets. The mere existence of corporate taxes, as recognised by Modigliani and Miller in their 1963 article, makes debt so much more attractive. A firm, faced with a positive corporate tax rate, would have the incentive to adopt a capital structure comprising 100% of debt. Other ‘frictions’, such as the cost of financial distress, information asymmetries and agency costs, as well as managerial motives, swing the balance back away from debt. These considerations, as usefully summarised by a number of textbooks (e.g. Arnold 2005), suggest that, depending on the circumstances of the individual investment opportunities faced by a firm, there may be a financial structure that minimizes the cost of capital and thus, given the expected cash flows, maximizes the value of the firm. Ultimately, this goes some way towards explaining the capital structure decisions made by firms in the real world.

**Weighted average cost of capital (WACC)**

The first practical problem that one encounters in implementing this perspective is the calculation of the cost of capital. For a given project, this is given by the proportion of debt and equity that the project absorbs and by their respective cost:

\[
k = \frac{k_d D}{D+E} + \frac{k_e E}{D+E}
\]

Here, \(k\) denotes the weighted average cost of capital (WACC), \(k_d\) denotes the cost of debt, \(k_e\) denotes the cost of equity and \(D\) and \(E\) denote the market value of debt and equity, respectively. The formula above gives the WACC for a single project or for the whole firm. This is because we can always see the firm as a big project comprising a number of subprojects, i.e. a portfolio of projects. Now, how do we calculate all these inputs for the formula? This is where it becomes important to understand the financial theory behind the models of the cost of capital. The cost of each form of capital, i.e. debt, equity or hybrids, is
the expected return demanded by investors to hold securities issued by the firm to raise that capital.

**Cost of debt capital**

In the case of debt capital, the cost is the yield of the debentures issued by the firm. The yield is also known as the internal rate of return (IRR) and it is the discount rate that equals the discounted present value of the cash flows of the debenture to the market price of the latter, i.e. such that the net present value (NPV) is zero. For example, for a bond maturing in 3 years with a €100,000.00 face value that trades at 95% of the latter, i.e. with a market price $P_0 = €95,000.00$, and pays a 5% annual coupon, $k_d$ is the rate such that:

$$95,000 = 5,000/(1+k_d)^1 + 5,000/(1+k_d)^2 + 105,000/(1+k_d)^3$$

The discount rate $k_d$ that solves this is about 6.9% per annum. It has to be calculated iteratively by trial and errors. Alternatively, we can try and formulate two educated guesses that give us a positive and negative NPV, respectively, and then resort to a linear interpolation between their values. For example, if we use as a first guess $k_{d1} = 6\%$ we get an NPV of

$$NPV = €[5,000/(1+6\%)^1 + 5,000/(1+ 6\%)^2 + 105,000/(1+ 6\%)^3] - €95,000 = €2,327.00$$

This is greater than zero, so as our second guess we try a larger discount rate, say $k_{d1} = 8\%$. This gives us:

$$NPV = €[5,000/(1+8\%)^1 + 5,000/(1+ 8\%)^2 + 105,000/(1+ 8\%)^3] - €95,000 = - €2,731.00$$

We now have a negative figure for NPV so, to get an approximation of the IRR, we interpolate the two guess discount rate as follows:

$$k_d \equiv k_{d1} + [NPV_{d1}/(NPV_{d1} - NPV_{d2})] (k_{d2} - k_{d1})$$

$$= 6\% + €[2,327/(2,327 - (-2,731))] (8\% - 6\%)$$

$$= 6.92\%$$

The biggest practical problem is, however, not the calculation but rather the fact that there is often considerable uncertainty around the price of corporate debentures as, with the exception of bonds issued by large and high-profile corporations, they are often infrequently traded or at least they are traded in relatively illiquid markets. The traded price of the debenture, for the purpose of the calculations above, is the price at which it can or could be exchanged in a market transaction as, from the point of view of an investor interested in maximizing her own wealth, this gives the opportunity cost of the funds tied up in the debenture itself. In the case of actively traded bonds, this is simply their market price (the dirty price, i.e. clean price + accrued interest).

As far as loans are concerned, these are not typically traded in an open secondary market, at least not directly\(^1\). Like all securities, however, they have a ‘primary market’. That is, we can always ask a few competing banks for a quotation on a loan corresponding, with

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\(^1\) Credit derivatives, when traded in a well functioning market, offer a way of indirectly pricing the default risk embedded in corporate debt.
respect to quantity and characteristics (maturity, covenants, etc) to the amount of debt capital that we need to price for the purpose of our calculations. When the traded price is 100% of the face value, the yield at inception is the interest rate. This is the case of one of the most common types of loans, those with a “bullet” structure, i.e. loans that pay periodic interest on 100% of the initial face value until maturity with no amortization of capital due. In this very simple case, the interest quoted would give us the cost of debt capital $k_d$.

For example, we might ask a number of banks for a quotation on a large loan for firm ABC. The banks, after a thorough credit-worthiness review, might get back to us with offers of funds at a spread of, say, 3% per annum over the annualised 3 Month Euribor rate (a popular benchmark interest rate). If the annualised Euribor rate was, 2% per annum, it would mean that the firm if being offered a loan at a rate of 5% = 35 + 2% per annum. This could be taken as the cost of debt capital for the firm at that particular point in time. A particularly simple case is represented by bullet floating rate notes or loans. On condition that no change in the credit risk of the issuer has intervened since inception or issuance, their price can be taken to be 100% of the face value immediately after the payment of each coupon or period interest and very close to 100% at all other times. In this special case, the yield at any one time (and not just at inception) coincides with the contractual interest rate. In other more elaborate cases, we would have to perform dedicated IRR calculations.

Cost of equity capital
In the case of equity capital, the cost is the expected return demanded by investors to hold the equity issued by the firm. The equity could be held by either a relatively small group of private investors who have invested a relatively large portion of their wealth in the company or by a large number of small investors, each of whom have invested only a small fraction of their wealth in the company. The former case is common with small private firms whereas the latter is typical of relatively large firms run as public companies listed on the stock market. Let us refer to the first type of investors as ‘the entrepreneur’ and to an investor who belongs to the second group as the ‘representative stock market investor’. In between these two extremes, there are obviously a number of intermediate situations.

The famous Capital Asset Pricing Model (CAPM) works best to describe the return expected, for example by stock market investors, on investments that represent a relatively small portion of a diversified portfolio. It is only in this circumstance that the diversification argument that underpins the CAPM truly holds. The CAPM best known equation is perhaps the so called Security Market Line (SML). This says that the expected return on a risky security is given by the sum of the risk free rate $r_f$ plus a risk premium that is proportional to the amount of non-diversifiable or systematic risk, the so called $\beta$ (beta) coefficient, of the security:

$$k_e = r_f + (k_m - r_f) \beta$$

Here, $k_m$ is the expected return on a particular risky asset, often known as the “market portfolio”, i.e. the market capitalization-weighted portfolio of all risky assets, or at least of all the risky securities held by the representative stock market investor. Often, diversified stock market indices such as the S&P500 are used as a proxy for the market portfolio of risky assets and their realised average return over long periods of time (say, 50 years) are taken as an estimate of $k_m$.

Risk free rate
As far the risk free rate $r_f$ is concerned, a good proxy would be the yield on default free sovereign debt (such as German Bunds and US treasuries) of duration comparable to the
investment horizon of the project being evaluated or, when calculating the WACC of a firm, to the average duration of the firm’s debt (for the definition of duration, see Arnold (2005) or Copeland and Weston and Shastri (2005)). To get an idea of its order of magnitude, the average real risk-free rate for the period calculated by Fama and French (2002) is 2.19% per annum.

**Equity premium**
The level of \( k_m \) in excess of the risk-free rate is the so called equity premium, i.e. \( k_m - r_f \). Earlier estimates of this crucial quantity were in the region of 6% per annum in real terms but more recent estimates put the same figure more in the region of 4.5% per annum. The basis of this latter estimate is Gordon’s Discounted Dividend Growth Model (DDM) applied to a diversified stock market index. According to this model, the price \( P \) of the market portfolio is given by

\[
P_0 = D_1/(k_m - g_d)
\]  

(3)

Here, \( D \) is the next declared aggregate dividend (the value-weighted average of the dividends on all stocks included in the index), \( k_m \) is the discount rate and \( g_d \) is the expected growth rate of \( D \), which is assumed to be constant in the long run. Given this latter assumption and the further assumption of constant payout rate, this formula can be rewritten with earnings growth \( g_e \) in place of dividend growth:

\[
P_0 = D_1/(k_m - g_e)
\]  

(4)

Taking natural logs of both sides, we obtain the following approximation:

\[
p = d - k_m + g_e
\]  

(5)

Here, the lower case letters \( p \) and \( d \) denote the natural logs of \( P_0 \) and \( D_1 \). The above expression can be conveniently rewritten to give \( k_m \)

\[
k_m = d - p + g_e = (d - p) + g_e
\]  

(6)

The expression \( d - q \) is the (log, i.e. continuously compounded) dividend-yield which, in the calculations of Fama and French (2002), averages 3.7% per annum. The same authors use the historical average of dividend growth on S&P500 stocks to proxy \( g_e \). Over the period 1950-2000, this figure stands 4.74% per annum in real terms. Plugging into (6), we thus have:

\[
k_m = 3.7\% + 2.82\% = 6.52\%
\]  

(7)

In the calculations of Fama and French (2002), the average real risk-free rate for the period is 2.19% per annum. We thus have an equity risk premium, i.e. the excess of the market portfolio expected return over the risk free rate, equal to 4.33% = 6.52% – 2.19% per annum in real terms over the period 1950-2000. This is lower than the average market return over the same period, which is about 6% per annum. In financial analysts valuations, the lower equity premium estimate based on the approach followed by Fama and French (2002) has now replaced the higher estimate based on the raw historical average, which is considered implausibly high also on the basis of other fundamental theoretical considerations and is deemed to be probably due to a once-off run of good luck in stock markets (that culminated in the so called ‘dot-com bubble’) rather than a reflection of ex-ante investors expectations.
about equity returns. The difference between the Fama and French (2002) and the raw historical average estimate shows the importance of applying sound financial theory in refining our inputs into the WACC formula and into financial calculations in general. Similar results are obtained for most developed stock markets and sample periods that extend to recent times.

**Beta**

In the CAPM, the beta of an asset is formally defined as a regression coefficient. More specifically, if we use the term excess-return to refer to the return in excess of the return on the risk-free rate, the beta is the coefficient of the regression of the excess-return on the asset on the excess-return on the market portfolio. At a more intuitive level, viewing the latter as a ‘factor’ or the main driver of changes in the former, beta is the coefficient that quantifies how sensitive the excess-return on the asset is to changes to the market excess-return.

This coefficient can be estimated by running the required regression when data on the asset excess-return are available. This is typically only the case of listed stocks. For shares of companies that are not listed, we might use as an indication the beta of a stock of a firm with similar operations, so called “comparables”. Because beta is a function of both operating leverage and financial leverage, we need to first “un-lever” the beta of the comparable if the financial leverage of the latter is different from the financial leverage of the shares that we are attempting to value. The formula to calculate a company’s unlevered beta is:

$$\beta_U = \frac{\beta_L}{1 + (1 - T_C) \frac{D}{E}}$$

(8)

Where:

- $\beta_L$ is the comparable firm’s beta with leverage
- $T_C$ is the corporate tax rate of the comparable
- $\frac{D}{E}$ is the company’s debt/equity ratio

For example, suppose we know that a ‘comparable’ has a beta equal to 0.95, a debt/equity ratio of 50% and a 10% tax rate. Its unlevered beta is then:

$$\beta_U = \frac{0.95}{1 + (1 - 0.1) 0.5} \approx 1.73$$

(9)

If the comparable is truly similar to the company we are interested in, we can take the unlevered beta of the former as the unlevered beta of the latter. This can be used directly to calculate the WACC of the firm under consideration.

**Putting it all together**

For example, in the case of the firm considered above, we can plug the estimated unlevered beta, real risk-free rate and real equity premium into the SML equation of the CAPM:

$$k_e = r_f + (k_m - r_f)\beta = 2.19\% + (4.33\%) \times 0.65 = 5.00\%$$

(10)

Thus, we get an estimated real cost of capital for this firm of 5.00% per annum. This can be taken as an estimate of the WACC (not as the cost of equity of the company that owns the firm, because to get this we should have used a levered beta, i.e. we should have re-levered beta to take into account the financial leverage of the company). This is a figure in
real terms. To get the figure in nominal terms the safest approach is to use a nominal equity premium and nominal risk-free rate (i.e., the real equity premium and the real risk-free rate, respectively, plus the inflation rate) in place of the real rates.

Other excellent examples of how to calculate the equity expected return using the CAPM, as well as on how to use this calculation to calculate the WACC of a firm or project and use the calculated WACC for valuation purposes, are provided in the articles authored by past examiners published in recent years.

**Capital structure**

It is important to understand that all these calculations implicitly assume a given ratio of debt to equity, i.e. a given capital structure.

**Objectives of the firms and valuation**

It is at this point important to pause for a minute and reflect on what is the purpose of our analysis and the overall aim. We may presume that the objective is to maximize the value of the firm, as this takes into consideration the interests of both bondholders and shareholders. Alternatively, we might seek or be asked to maximize shareholders’ wealth. In a perfect capital market, maximizing firm value and maximization of shareholders wealth would never be conflicting goals, but in more realistic circumstances they might be. For example, if management ends up using loaned funds to make investments that are riskier than what debt holders had anticipated and priced in, this would shift wealth from debt holders to shareholders and, in so doing, it might maximize shareholders’ value while not maximizing the value of the firm. This would be an instance of debt-holders ‘expropriation’, which in turn is but one example of the problems that arise as a result of the information asymmetries and agency costs that plague the agency relation between ‘insiders’ (e.g., shareholders, managers) and ‘outsiders’ (debt-holders, minority shareholders, etc.) and it is the main reason why capital structure and risk management, against the prediction of Modigliani and Miller famous ‘first proposition’, does matter.

**References**

